

On the Casimir Effect in the High Tc Cuprates

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Abstract

High temperature superconductors have in common that they consist of parallel planes of copper oxide separated by layers whose composition can vary. Being ceramics, the cuprate superconductors are poor conductors above the transition temperature, T_c . Below T_c , the parallel Cu-O planes in those materials become superconducting while the layers in between stay poor conductors. Here, we ask to what extent the Casimir energy that arises when the parallel Cu-O layers become superconducting could contribute to the superconducting condensation energy. Our aim here is merely to obtain an order of magnitude estimate. To this end, the material is modelled as consisting below T_c of parallel plasma sheets separated by vacuum and as without a significant Casimir effect above T_c . Due to the close proximity of the Cu-O planes the system is in the regime where the Casimir effect becomes a van der Waals type effect, dominated by contributions from TM surface plasmons propagating along the ab planes. Within this model, the Casimir energy is found to be of the same order of magnitude as the superconducting condensation energy.

1 Introduction

The question has remained open how Cooper pairs can be stable at around 100K where some high temperature superconductors (HTSCs) are still superconducting. In particular, the phonon-mediated attractive electron-electron interaction of conventional superconductors [1] is too weak at these temperatures. While much is known about the microscopic mechanism of high temperature superconductivity, for example, that it involves d -wave Cooper pairs [2, 3], the energetics that drives this mechanism is still unclear [4]. For a new approach to the question of the energetics behind high temperature superconductivity, let us reconsider a feature that HTSCs have in common, namely parallel superconducting layers which are separated by layers of essentially insulating material. Since in between any two conducting planes there occurs a Casimir effect [5, 6, 7], the effect should also occur between the parallel superconducting layers in HTSCs, as was first pointed out in [8].

Before estimating the significance of the Casimir effect in HTSCs, let us recall the textbook case of two parallel plates that are separated by vacuum

and that are ideal conductors, i.e., conductors whose conductivity and Meißner effect expel electromagnetic fields of all wavelengths with vanishing penetration depths. If the plates' area, A , is large compared to their distance, a , the Casimir energy reads

$$E_c(a) = -\frac{\pi^2 \hbar c A}{720 a^3}, \quad (1)$$

thus leading to an attractive force. Corrections that take into account the finite conductivity of real metals have been calculated for geometries such as parallel plates and a plate and a sphere, along with corrections for finite surface roughness and finite temperature, see [9]. Recent experiments measured the Casimir force between a metallic plate and sphere down to distances of around $100nm$, confirming the theoretical predictions with a precision of 0.5% [10].

In the case of HTSCs, as the temperature is lowered below T_c , superconducting charge carriers form in parallel layers. The onset of superconductivity does not make these Cu-O layers ideal conductors but even only as superconductors these layers should lead to some extent to a Casimir effect and therefore to some negative Casimir energy. If this lowering of the energy at the onset of the superconductivity is large enough then it could be the very reason why the Cu-O layers' initially non-superconducting charge carriers are able to form superconducting charge carriers. Because of the Casimir effect which arises at the onset of superconductivity, the initially non-superconducting charge carriers would be energetically driven to use whichever microscopic mechanism is available to them to form superconducting charge carriers. In this scenario, it would therefore be the very effects of superconductivity which enable and stabilize superconductivity. Cooper pairs would derive their stability collectively, across layers. Namely, Cooper pairs would be stable because if sufficiently many of the Cooper pairs on opposing layers were to break up then the Casimir effect would cease and the energy would have to go back up.

2 Estimating the size of the Casimir effect in HTSCs

Our aim now is to estimate if the Casimir effect in HTSCs could indeed be sufficiently large to make the formation of Cooper pairs energetically favorable at temperatures as high as 100K. Clearly, the superconducting Cu-O layers are much less efficient at suppressing electromagnetic fields than ideal conductors would be, and the more so the shorter the wavelength. In particular, since the layer spacing is two to three orders of magnitude smaller than for example the London penetration depth, the Casimir effect should be suppressed by several orders of magnitude. In order to estimate the order of magnitude of the actually occurring Casimir effect in HTSCs, let us crudely model the charge carriers of the superconducting Cu-O planes in the superconducting state as forming 2-dimensional parallel plasma sheets separated by vacuum. For the normal state we make the approximation that the material does not possess a significant Casimir effect.

It is known that for two parallel plasma sheets with large separation, a , the Casimir energy as a function of a is given by Eq.1. The Casimir energy for parallel plasma sheets in the regime of small a has recently been calculated, [11]:

$$\tilde{E}_c(a) = -5 \cdot 10^{-3} \hbar c A a^{-5/2} \sqrt{\frac{nq^2}{2mc^2\epsilon_0}} \quad (2)$$

In our model we are of course in the regime of high layer transparency, i.e., in the small a regime where Eq.2 can be applied. Applying Eq.2 to our situation here, with a typical layer separation of $a = 1nm$, we obtain a Casimir energy which is suppressed by four orders of magnitude compared to what the Casimir energy would be for two ideally conducting layers. For example, we obtain $\tilde{E}_c(1nm)/E_c(1nm) = 4.3 \cdot 10^{-4}$ when using for the charge carriers' area density, charge and mass the realistic values $n = 10^{14}(cm)^{-2}$, $q = 2e$ and $m = 2\alpha m_e$ with $\alpha = 5$. Let us now identify $\tilde{E}_c(a)$ with the condensation energy

$$E_{cond.} = \tilde{E}_c(a) \quad (3)$$

and calculate the corresponding transition temperature T_c . First, we have

$$T_c = \Delta(0)/\eta k_B \quad (4)$$

where for HTSCs the parameter η is generally thought to be around or somewhat larger than the BCS value of $\eta = 1.76$. Next, we relate the gap energy to the density of states and to the condensation energy through:

$$E_{cond} = -D(\epsilon_F)\Delta^2(0)/2 \quad (5)$$

The density of states in the case of a Fermi gas in two dimensions reads $D(\epsilon_F) = mA/\pi\hbar^2$, so that we finally obtain this prediction for the transition temperature:

$$T_c(a) = \frac{2^{1/4}\pi^{1/2}\hbar^{3/2}e^{1/2}n^{1/4}}{10\eta k_B m^{3/4}\epsilon_0^{1/4}a^{5/4}} \quad (6)$$

Notice that, curiously, T_c is a function of n/a^5 . If we choose realistic values such as $a = 1nm$, $n = 10^{14}(cm)^{-2}$, $\eta = 1.76$ and $m = 5m_e$, this yields:

$$T_c = 125K \quad (7)$$

In order to illustrate how strong the Casimir effect would be at such small layer distances, if the Casimir effect were not suppressed because of the high layer transparency, let us compare with the result that is obtained when modelling the Cu-O layers as ideally conducting sheets. In this case, Eq.1 would apply instead of Eq.2 which would have led to a prediction of a transition temperature of $T_c = 3350K$. The result for T_c given in Eq.7 is of course only a very rough estimate, but it shows that the Casimir effect could be of the right order of magnitude. Within our simple model, even though the Casimir effect is highly suppressed in HTSCs, the small Casimir effect that does occur could still be large enough to account for the superconducting condensation energy in HTSCs.

3 Conclusions

Within this scenario, the stability of Cooper pairs is a collective phenomenon as it involves the Casimir effect across layers. More precisely, it is mostly a van der Waals type effect between the Cu-O layers: because of the small distance between neighboring layers the retardation of the electromagnetic interaction which couples the charge and current fluctuations on one Cu-O layer to the fluctuations on nearby Cu-O layers are negligible and we are therefore in the regime where the Casimir effect becomes a van der Waals type effect. Correspondingly, c did in fact drop out of our calculations. Therefore, in this scenario, the energetic gain at the onset of superconductivity is due to an increase in the fluctuations along the Cu-O layers that are in sync with fluctuations on nearby Cu-O layers, which then leads to an increased attractive van der Waals type force in between the Cu-O planes - with a corresponding drop in a van der Waals type interaction energy. We can say somewhat more because it is known that the Casimir effect for parallel plasma sheets is at short distances dominated by contributions from TM surface plasmons propagating along the plasma sheets [11]. Applied to our scenario, this would imply that the fluctuations that are most enhanced after the onset of superconductivity and that therefore contribute most to the energetics of the Casimir van der Waals effect are the TM “surface” plasmons propagating along the superconducting Cu-O layers. These TM surface plasmons should presumably also play a role in the microscopic mechanism that allows Cooper pairs to form. We recall that the pairing interaction is today expected to be of magnetic origin [2].

4 Outlook

In order to make testable quantitative predictions, our model needs to be significantly improved in several respects. First, it will need to be taken into account that also above T_c the cuprates possess to some extent a layered pattern of (small) conductivity, probably describable by a Drude model, leading to a small Casimir energy. The Casimir energy that is available as condensation energy is of course only the difference between the Casimir energies above and below T_c .

Also, we considered so far only the case of two layers. Actual HTSCs possess not only many layers but also multiple layer distances within and among the units cells. In principle, it should be straightforward to generalize our present results for multiple layers at multiple distances, namely by calculating the transmission and reflection coefficients of more layers from those of fewer layers, iteratively, and if need be numerically. Given the high transparency of the Cu-O layers also the coupling between fluctuations of layers whose distance may not be in the short distance regime may need to be taken into account, in which case also some Casimir effect of non van der Waals origin would play a role. These calculations should then offer an opportunity to check our scenario experimentally.

For example, in $YBa_2Cu_3O_{7-x}$ (YBCO), which becomes superconducting

at around $92K$ the crystallographic unit cell contains two copper oxide layers at a distance of $a_b \approx 0.39nm$ and neighboring such bi-layers are separated by a layer of essentially nonconducting material of width $a_i \approx 1.17nm$. The area density of the superconducting charge carriers on each Cu-O layer is roughly on the order of $n \approx 10^{17}/m^2$. For our purpose, the case of YBCO is of particular interest because of the availability of experimental data, [12, 13], on epitaxial superlattices in which slabs of YBCO alternate with slabs of insulating material, namely $PrBa_2Cu_3O_{7-x}$ (PrBCO). For example, in the experiments reported in [12], the authors varied the thickness of the YBCO slabs from $M = 1$ to $M = 8$ unit cells and the thickness, a_m , of the PrBCO slabs from $N = 1$ to $N = 16$ unit cells, i.e., in the range $a_m = 2nm$ to $20nm$. The superconducting transition temperature was measured as a function, $T_c(N, a_m)$, of N and a_m and it was found that T_c characteristically increases with decreasing layer distances. Since our ansatz predicts a dependence of T_c on the layer separation, Eq.6, those data should provide a good testing ground for the present ansatz. Given that the Casimir effect diminishes with increasing layer distances our approach should match the experimental data at least qualitatively. Work in this direction is in progress.

If our general scenario applies, it would mean that in order to raise T_c the challenge would be to create layered materials, not necessarily cuprates, for which the Casimir effect in the superconducting state is as large as possible. In order to maximize the Casimir energy available as condensation energy the same material would also have to possess an as small as possible Casimir effect in the normal state. Of course, for the Casimir effect to enable superconductivity energetically, the material in question must first possess some microscopic mechanism that allows superconducting charge carriers to form in layers. Let us consider, for example, carbon nanotubes (CN) as these can be made superconducting [14]. Within our scenario, it is to be expected that, due to the Casimir effect, T_c should be higher for multi-walled than for single-walled CNs. Indeed, recently, a 30-fold increase in T_c for multi-walled CNs has been reported [15].

It should also be interesting to explore a possible connection to the approach in [16]. There, the condensation energy has been suggested to arise from an increased screening of the charge carriers' Coulomb repulsion at the onset of superconductivity, an effect which is also characteristically layer-distance dependent.

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References

- [1] J.R. Schrieffer, *Theory of Superconductivity*, Benjamin Inc., New York, N.Y. (1964)
- [2] J.R. Schrieffer, J.S. Brooks, *Handbook of High-Temperature Superconductivity*, Springer Verlag, New York (2007)
- [3] M. Tinkham, *Introduction to Superconductivity*, Dover Publications, Mineola, N.Y. (1996), P.W. Anderson, *The Theory of Superconductivity in the High- T_c Cuprates*, Princeton University Press, Princeton, N.J. (1997), Y. Yanase et al, Phys. Rept., **387**, 1 (2003)
- [4] A.J. Leggett, J. Phys. Chem. Solids **59**, 1729-32 (1998)
- [5] H.B.G. Casimir, D. Polder, Phys. Rev. **73**, 360 (1948)
- [6] V. Mostepanenko, N. N. Trunov, *The Casimir Effect and Its Applications*, Oxford University Press (1997)
- [7] K.A. Milton, *The Casimir Effect*, World Scientific, Singapore (2001)
- [8] A. Kempf, in Proc. 10th Marcel Grossmann meeting, Rio de Janeiro, 20-26 July 2003, Eds. M. Novello, S.P. Bergliaffa, R. Ruffini, World Scientific, Singapore, part C, 2271 (2005), gr-qc/0403112
- [9] G. Barton, N.S.J. Fawcett, Phys. Rept. **170**, 1 (1988), V.B. Bezerra, G.L. Klimchitskaya, V.M. Mostepanenko, Phys. Rev. **A62**, 014102 (2000), M. Bordag, U. Mohideen, V.M. Mostepanenko, Phys. Rept. **353**, 1 (2001), Y. Aghababaie, C.P. Burgess, Phys. Rev. **D70**, 085003 (2004), S.K. Lamoreaux, Rep. Prog. Phys., **68**, 201 (2005)
- [10] R.S. Decca et al., Annals Phys. **318** 37 (2005), quant-ph/0503105, U. Mohideen, A. Roy, Phys. Rev. Lett., **81**, 4549 (1998)
- [11] M. Bordag, J. Phys. **A39**, 6173 (2006)
- [12] D.H. Lowndes, D.P. Norton, J.D. Budai, Phys. Rev. Lett. **65**, 1160 (1990)
- [13] Q. Li, X.X. Xi, X.D. Wu, A. Inam, S. Vadlamannati, W.L. McLean, Phys. Rev. Lett. **64**, 3086 (1990)
- [14] Z.K. Tang et. al., Science, **292**, 2462 (2001), M. Kociak et. al., Phys. Rev. Lett. **86**, 2416 (2001)
- [15] I. Takesue, et. al., Phys. Rev. Lett. **96**, 057001 (2006)
- [16] A. J. Leggett, Proc. Natl. Acad. Sci. USA, **96**, 8365 (1999)